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Technical Note

Vortex motion influencing sphere heating—Reynolds analogy revisited

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Abstract

Recently, it was reported [M. Masoudi, W.A. Sirignano, Influence of an advecting vortex on the heat transfer to a liquid droplet, Int. J. Heat Mass Transfer 40 (15) (1997) 3663–3673] that when sphere heating in a uniform flow is perturbed by vortex motion, global self-similarity is observed and the resulting correlation predicts that the sphere Nusselt number fluctuations due to vortex motion scale with the vortex circulation with unity exponent: $Nu' \sim \Gamma_0/(2\pi)$ (Nu' = perturbation in sphere Nusselt number, Γ_0 = non-dimensionalized initial vortex circulation). It is shown here that this computational observation is also obtained using Reynolds analogy. © 1999 Elsevier Science Ltd. All rights reserved.

1. Discussion

A widely used correlation [1] predicts that the heating of a cold sphere in a hot gas stream follows

$$Nu_{\text{ax}} = 1 + (1 + Pr\,Re)^{1/3}\,Re^{0.077} \tag{1}$$

in the range $1 < Re < 400$, $0.25 < Pr < 100$. This correlation is however applicable only when the sphere heating takes place in an axisymmetric (uniform) flow.

To observe the effect of vortical perturbations as well as to gain insight into sphere heating in an asymmetric flow, three-dimensional interactions between a cold sphere embraced in a hot uniform gas stream and a Rankine vortex advecting in the gas phase were simulated using full Navier–Stokes and energy equations [2]. The numerical experimentation demonstrated that the sphere Nusselt number in a uniform flow perturbed by vortex motion is predicted by

$$\frac{\overline{Nu_{\text{asym}}}}{\overline{Nu_{\text{ax}}}} - 1 = 0.019\frac{\Gamma_0}{2\pi}Re^{0.40}\,\tanh\left(0.50\frac{d_0}{\sigma_0^{0.6}}\right). \tag{2}$$

Here, $\overline{Nu_{\text{ax}}}$ is the same as Nu_{ax} in Eq. (1) and $\overline{Nu_{\text{asym}}}$ is the time-averaged Nusselt number in the perturbed flow $Nu_{\text{asym}}(t)$. It is therefore fair to denote the left-hand-side of this equation $\overline{Nu_{\text{asym}}}/\overline{Nu_{\text{ax}}} - 1 \equiv Nu'$ with Nu' being the net perturbation in sphere Nu. Note that here the base-flow is 'perturbed' by the motion of the vortex, i.e. the vortex is not strong enough to reverse the base-flow.

When a vortex is relatively far from the sphere, it appears to the sphere as a point vortex. Therefore, Eq. (2) [with $d_0/\sigma_0^{0.6} \gg 1$, thus, $\tanh(\;) \to 1$] states that the Nusselt number variations are proportional to the vortex circulation

$$Nu' \sim \frac{\Gamma_0}{2\pi}. \tag{3}$$

There is a rather surprising observation here—it appears that the sphere heating continually depends on the base flow Reynolds number with non-unity exponents [Eqs. (1), (2); Nu_{ax}, $Nu' \sim Re^m$, $m \neq 1$]. However,

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Nomenclature

a	droplet radius (characteristic length)
d_0	initial vortex location (non-dimensionalized by a^*)
Nu	Nusselt number
Pr	Prandtl number
Re	$=U_\infty^*(2a^*)/\nu^*$, base-flow Reynolds number
Re_v	$=v_{\mathrm{max},0}^*(2\sigma_0^*)/\nu^*$, vortex Reynolds number
St	$=Nu/(Re\ Pr)$, Stanton number
U_∞	far upstream gas field velocity (characteristic velocity)
v_max	vortex maximum tangential velocity (non-dimensionalized by U_∞^*).

Greek symbols

Γ_0	$=2\pi\sigma_0 v_{\mathrm{max},0}$, non-dimensionalized initial vortex tube circulation
ν	gas kinematic viscosity
σ	radius of vortex tube (non-dimensionalized by a^*).

Subscripts

asym	quantity in the asymmetric flow (i.e. with vortex)
ax	quantity in the axisymmetric (uniform) flow (i.e. with no vortex)
v	vortex quantity
0	initial quantity.

Superscript

*	dimensional quantity
‾	time-averaged quantity.

the sphere heating depends on the vortex circulation with a unity exponent [Eq. (3)]. There seems to exist no explanation for this shift in exponents.

The goal here is two-fold. First, the rigorous computational observation resulting in the role of vortex circulation Γ_0 in Nu' (Eq. (3)) is shown to be also predicted using Reynolds analogy. Second, Reynolds analogy provides insight into the unity exponent of the vortex circulation.

Reynolds analogy, which relates fluid flow to heat transfer, yields the following for the heating of a sphere in an axisymmetric flow [3]

$$St\sqrt{Re_x} = 0.762 Pr^{-0.6}.$$

We substitute for $\sqrt{Re_x}$ with $\sqrt{Re}$ and therefore, for Nu with $\overline{Nu}_{ax}$ in $St=Nu/(Re\ Pr)$. (This substitution is justified since, analogous to Eq. (2), the Nusselt number is averaged first over the sphere surface and then in time, so that here '‾' represents an averaging first in space, then in time.) The following can then be derived for sphere heating in an axisymmetric flow:

$$\overline{Nu}_\mathrm{ax} \sim Re^{0.5}\ Pr^{0.4}. \tag{4}$$

In an axisymmetric flow, the dynamic interaction is merely due to the base-flow and is represented by Re. However, when the base-flow is perturbed by vortex motion, the interaction is not merely due to the base-flow, but due to a perturbed one: $Re+Re_\mathrm{v}$. Using this in lieu of Re in Eq. (4) and therefore, replacing $\overline{Nu}_\mathrm{ax}$ with $\overline{Nu}_\mathrm{asym}$, expanding using the Taylor series, deducting Eq. (4) from the result, dividing through by Eq. (4), and denoting $\overline{Nu}_\mathrm{asym}/\overline{Nu}_\mathrm{ax} - 1$ by Nu' as before, one derives

$$Nu' \sim \frac{Re_\mathrm{v}}{Re} \tag{5}$$

but

$$\frac{Re_\mathrm{v}}{Re} = \frac{v_{\mathrm{max},0}^*}{U_\infty^*}\,\frac{(2\sigma_0^*)}{(2a^*)} = v_{\mathrm{max},0}\,\sigma_0 = \frac{\Gamma_0}{2\pi} \tag{6}$$

therefore,

$$Nu' \sim \frac{\Gamma_0}{2\pi} \tag{7}$$

which is the same as Eq. (3). Two observations are made here. First, and foremost, the functional dependency of the sphere Nusselt number perturbation Nu' on the vortex circulation Γ_0, which was derived through numerical experimentation (Eq. (3), Ref. [2]), is confirmed by Reynolds analogy. Second, in this dependency, the unity exponent in Γ_0 is the consequence of the relative significance of the vortex

Reynolds number to the base-flow Reynolds number: Re_v/Re.

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